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Mark Scheme (Results)

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In Further Pure Mathematics (4PM1)
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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme.

  Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.
- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)
- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**

  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**

  You must always check the working in the body of the script (and on any diagrams) irrespective of whether the final answer is correct or incorrect and award any marks appropriate from the mark scheme.
  If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**

  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg incorrect cancelling of a fraction that would otherwise be correct.

  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect: eg algebra.

  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**

  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right),\text{ where }|pq|=|c|$$ leading to $x=\ldots.$

$$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)\text{ where}|pq|=|c|\text{ and }|mn|=|a|$$ leading to $x=\ldots.$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\ldots.$

3. Completing the square:

$$x^2+bx+c=0:\ \left(x\pm\tfrac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$$ leading to $x=\ldots.$

4. Use of calculators

Unless the question specifically states 'show' or 'prove' accept correct answers from no working. If an incorrect solution is given without any working do not award the Method mark.

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\to x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\to x^{n+1}\right)$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is <u>not</u> quoted, the method mark can be gained by implication

from the substitution of <u>correct</u> values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers <u>may</u> be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this: eg in a case of "prove or show....")

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)

Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

Multiple attempts at a question.

If there is a choice of methods shown, then award the lowest mark, unless the subsequent working makes clear the method that has been used.

Question number	Scheme	Marks
1	$\alpha+\beta=\frac{3}{4},\ \alpha\beta=-2$ Sum: $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{\frac{3}{4}}{-2}=-\frac{3}{8}=\left(-\frac{b}{a}\right)$ Product: $\frac{1}{\alpha}\times\frac{1}{\beta}=\frac{1}{\alpha\beta}=-\frac{1}{2}=\left(\frac{c}{a}\right)$ Equation: $x^2+\frac{3}{8}x+\left(-\frac{1}{2}\right)=0\Rightarrow 8x^2+3x-4=0$	B1B1 M1A1 B1ft M1A1ft [7]
		Total 7 marks

Mark	Notes
B1	For correct value for $\alpha+\beta$
B1	For correct value for $\alpha\beta$
M1	For the sum $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{their\frac{3}{4}}{their-2}$ Allow use of their stated $\alpha+\beta$ and $\alpha\beta$
A1	For the correct sum $-\frac{3}{8}$
B1ft	For the correct value of the product $\frac{1}{\alpha}\times\frac{1}{\beta}=\frac{1}{\alpha\beta}=-\frac{1}{2}$ FT their stated $\alpha+\beta$ and $\alpha\beta$
M1	For correctly forming an equation with their sum and product $x^2-\frac{'-3'}{8}x+-\frac{'1'}{2}(=0)$ Condone the absence of =0 for this mark.
A1ft	For the correct equation $8x^2+3x-4=0$ oe Follow through from their sum and product. Must be integer coefficients and constant.

Question number	Scheme	Marks
2	$(p-1)^2 - 4 \times 2 \times -2p > 0 \Rightarrow p^2 + 14p + 1 > 0$ critical values $p = \dfrac{-14 \pm \sqrt{14^2 - 4 \times 1 \times 1}}{2} = -7 \pm 4\sqrt{3}$ $p < -7 - 4\sqrt{3}$ OR $p > -7 + 4\sqrt{3}$	M1A1 M1 M1A1 [5]
		Total 5 marks

Mark	Notes
M1	Uses $b^2 - 4ac$ on the given quadratic equation with correct a, b, c; $a = 2$ $b = p - 1$ $c = -2p$ **and** a correct substitution to obtain $(p-1)^2 - 4 \times 2 \times -2p$ **Note:** Accept for this mark any inequality, equals sign and even $b^2 - 4ac$ used on its own.
A1	For the correct 3TQ with the correct inequality. **Note:** Allow ≥ or ≤ in place of > and < for this mark. $p^2 + 14p + 1 > 0$ or $-p^2 - 14p - 1 < 0$
M1	For an attempt to solve their 3TQ (provided it is a 3TQ) in terms of p by any acceptable method to obtain 2 values for p. See General Guidance for the definition of an attempt by factorisation, formula or completing the square. Use of calculators: it their 3TQ is incorrect, do not award this mark if working is not seen. $p = \frac{-14 \pm \sqrt{14^2 - 4 \times 1 \times 1}}{2} = -7 \pm 4\sqrt{3}$ oe Accept awrt -13.9, awrt -0.072
M1	For forming the correct inequalities with **their** critical values, provided they have been obtained from a 3TQ. $p < -7 - 4\sqrt{3}$ OR $p > -7 + 4\sqrt{3}$ oe ft their values from their $p^2 + 14p + 1 > 0$ or $-p^2 - 14p - 1 < 0$ **Note:** Condone use of AND for this mark. Condone ≥ or ≤ in place of > and < for this mark. Accept awrt -13.9, awrt -0.072
A1	For the correct inequality. $p < -7 - 4\sqrt{3}$ oe OR $p > -7 + 4\sqrt{3}$ oe **Note:** Must not indicate AND for this mark. Accept awrt -13.9, awrt -0.072

Question number	Scheme	Marks
3	Mark parts (i) and (ii) together	
	$\frac{r^2\theta}{2}=16.8\Rightarrow\left(\theta=\frac{33.6}{r^2}\right)$	B1
	$r\theta+2r=16.4$	B1
	$r\left(\frac{33.6}{r^2}\right)+2r=16.4\Rightarrow r^2-8.2r+16.8=0$	M1M1
	$r=\frac{8.2\pm\sqrt{(8.2)^2-4\times1\times16.8}}{2\times1}=4,\ (4.2)$	dM1A1
	$\theta=\frac{33.6}{'4'^2}=2.1$	dM1A1 [8]
	ALT	
	$r\left(\frac{33.6}{r^2}\right)+2r=16.4\Rightarrow 5r^2-41r+84=0\Rightarrow(5r-21)(r-4)=0$	[M1M1
	$\Rightarrow r=4,\ \left(\frac{21}{5}\right)$	dM1A1
	$\theta=\frac{33.6}{'4'^2}=2.1$	dM1A1]
	ALT– elimination of r by substitution	
	$\frac{r^2\theta}{2}=16.8$	[B1
	$r\theta+2r=16.4\quad\left(\Rightarrow\quad r=\frac{16.4}{(\theta+2)}\right)$	B1
	$\frac{\left(\frac{16.4}{(\theta+2)}\right)^2}{2}\theta=16.8$	M1 M1
	$33.6\theta^2-134.56\theta+134.4=0$	
	$\theta=2.1,(1.90\ldots)$	dM1A1
	$r=\sqrt{\frac{33.6}{'2.1'}}$ or $r=\sqrt{\frac{33.6}{'1.90\ldots'}}\Rightarrow r=4,\left(\frac{21}{5}\right)$	dM1A1
	ALT	B1
	$\frac{r^2\theta}{2}=16.8\Rightarrow\left(r=\sqrt{\frac{33.6}{\theta}}\right)$	B1
	$r\theta+2r=16.4$	M1
	$16.4=2\sqrt{\frac{33.6}{\theta}}+\theta\sqrt{\frac{33.6}{\theta}}\Rightarrow16.4=(2+\theta)\sqrt{\frac{33.6}{\theta}}$	M1
	$33.6\theta^2-134.56\theta+134.4=0$	dM1A1]

	$\theta = 2.1, (1.90\ldots)$ $r = \sqrt{\frac{33.6}{'2.1'}}$ or $r = \sqrt{\frac{33.6}{'1.90\ldots'}} \Rightarrow r = 4, \left(\frac{21}{5}\right)$	dM1A1
		Total 8 marks

Mark	Notes
B1	Uses the **correct** formula for the area of a sector to give $\frac{r^2\theta}{2} = 16.8$
B1	Uses the **correct** formula for the length of an arc to give $r\theta + 2r = 16.4$
M1	For attempting to eliminate θ by substitution: $r\left(\frac{33.6}{r^2}\right) + 2r = 16.4$ An attempt involves rearrangement of their linear equation to $\theta = \cdots$ followed by substitution into the area of a sector equation or rearrangement of their area of a sector equation to $\theta = \cdots$ followed by substitution into the linear equation. Allow if 2r omitted in their perimeter equation.
M1	For forming a 3TQ in *r* using only their expressions. $r^2 - 8.2r + 16.8 = 0$ or $5r^2 - 41r + 84 = 0$
dM1	For an attempt to solve their 3TQ to give **at least one** value of *r* See General Guidance for the definition of an attempt. This mark is dependent on the first M mark being awarded.
A1	For the correct value of *r*: $r = 4, (4.2)$ Reject $r = 4.2$ if given.
M1	For substituting *their r* into one of the two equations and rearranging to obtain θ
A1	For the correct value of θ. $\theta = 2.1$ Condone the value of θ that corresponds to 4.2 being included.
ALT – elimination of r by substitution	
M1	For attempting to eliminate *r* by substitution: $\frac{\left(\frac{16.4}{(\theta+2)}\right)^2}{2}\theta = 16.8$ or $16.4 = 2\sqrt{\frac{33.6}{\theta}} + \theta\sqrt{\frac{33.6}{\theta}}$ An attempt involves rearrangement of their linear equation to $r = \cdots$ followed by substitution into the area of a sector equation or rearrangement of their area of a sector equation to $r = \cdots$ followed by substitution into the linear equation. Allow if 2r omitted in their perimeter equation.
M1	For forming a 3TQ in θ using only their expressions. $33.6\theta^2 - 134.56\theta + 134.4 = 0$
dM1	For an attempt to solve their 3TQ to give **at least one** value of θ See General Guidance for the definition of an attempt. This mark is dependent on the first M mark being awarded.
A1	For the correct value of θ. $\theta = 2.1$ Condone the value of θ that corresponds to 4.2 being included.
M1	For substituting *their* θ into one of the two equations and rearranging to obtain *r*
A1	For the correct value of *r*: $r = 4, (4.2)$ Reject $r = 4.2$ if given.
	Note: In epen, award first A for r, second A for θ

Question number	Scheme	Marks
4	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\cos 2x\left(x^2-9\right)^{\frac{1}{2}}-\frac{1}{2}\times 2x\sin 2x\left(x^2-9\right)^{-\frac{1}{2}}}{\left(x^2-9\right)}$	M1A1A1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{2\cos 2x\left(x^2-9\right)^{\frac{1}{2}}\left(x^2-9\right)^{\frac{1}{2}}-\frac{1}{2}\times 2x\times\sin 2x}{\left(x^2-9\right)^{\frac{1}{2}}}}{\left(x^2-9\right)}\right\}$	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\left(x^2-9\right)\cos 2x-x\sin 2x}{\sqrt{\left(x^2-9\right)^3}}$ *	dM1A1 cso [5]
	ALT	
	$y=\sin(2x)\,(x^2-9)^{-\frac{1}{2}}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=2\cos(2x)(x^2-9)^{-\frac{1}{2}}+\sin(2x)\left(-\frac{1}{2}\right)(2x)(x^2-9)^{-\frac{3}{2}}$	[M1A1A1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\cos(2x)(x^2-9)-x\sin(2x)}{(x^2-9)^{\frac{3}{2}}}\right\}$	
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2(x^2-9)\cos(2x)-x\sin(2x)}{\sqrt{(x^2-9)^3}}$ *	dM1A1 cso]
		Total 5 marks

Mark	Notes
M1	For an attempt at Quotient rule. The definition of an attempt is that there must be a correct attempt to differentiate at least one term and the denominator must be $\left(\sqrt{x^2-9}\right)^2$. Allow the terms in the numerator to be the wrong way around, but the terms must be subtracted. Attempt at differentiation of the terms: $\sin(2x)\rightarrow k\cos(2x)$ where k is an integer $(x^2-9)^{\frac{1}{2}}\rightarrow lx(x^2-9)^{-\frac{1}{2}}$
A1	For correct differentiation of at least one term. $2\cos(2x)\,(x^2-9)^{\frac{1}{2}}$ or $-\frac{1}{2}\times 2x\sin 2x\,(x^2-9)^{-\frac{1}{2}}$
A1	For a fully correct Quotient rule $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\cos 2x(x^2-9)^{\frac{1}{2}}-\frac{1}{2}\times 2x\sin 2x(x^2-9)^{-\frac{1}{2}}}{(x^2-9)}$

dM1	For an attempt to rearrange to the given form. Dependent on M1 being scored. An attempt requires obtaining a single fraction and multiplying numerator and denominator by $(x^2-9)^{\frac{1}{2}}$ (see { } in mark scheme).
A1 **cso**	Fully correct method to show $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2(x^2-9)\cos 2x-x\sin 2x}{\sqrt{(x^2-9)^3}}$ Allow with $\sqrt{(x^2-9)^3}$ given as $(x^2-9)^{\frac{3}{2}}$ or $\sqrt{(x^2-9)}^3$
ALT – product rule	
M1	For an attempt at Product rule. The definition of an attempt is that there must be a correct attempt to differentiate at least one term. Attempt at differentiation of the terms: $\sin(2x)\left(x^2-9\right)^{-\frac{1}{2}} \to k\cos(2x)\left(x^2-9\right)^{-\frac{1}{2}}$ where k is an integer $\sin(2x)\,(x^2-9)^{-\frac{1}{2}} \to lx\sin(2x)\,(x^2-9)^{-\frac{3}{2}}$
A1	For correct differentiation of at least one term. Either $2\cos(2x)(x^2-9)^{-\frac{1}{2}}$ or $\sin(2x)\left(-\frac{1}{2}\right)(2x)(x^2-9)^{-\frac{3}{2}}$
A1	For a fully correct Product rule $\frac{\mathrm{d}y}{\mathrm{d}x}=2\cos(2x)(x^2-9)^{-\frac{1}{2}}+\sin(2x)\left(-\frac{1}{2}\right)(2x)(x^2-9)^{-\frac{3}{2}}$
dM1	For an attempt to rearrange to the given form. Dependent on M1 being scored. An attempt requires obtaining a single fraction and multiplying numerator and denominator by $(x^2-9)^{\frac{3}{2}}$ (see { } in mark scheme).
A1 **cso**	Fully correct method to show $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2(x^2-9)\cos 2x-x\sin 2x}{\sqrt{(x^2-9)^3}}$ Allow with $\sqrt{(x^2-9)^3}$ given as $(x^2-9)^{\frac{3}{2}}$ or $\sqrt{(x^2-9)}^3$

Question number	Scheme	Marks
5	$\log_3\sqrt{x-5}+\log_9(x+3)-1=0$	
	$\frac{1}{2}\log_3(x-5)+\frac{\log_3(x+3)}{\log_3 9}=1 \Rightarrow \left(\frac{1}{2}\log_3(x-5)+\frac{\log_3(x+3)}{2}=1\right)$	M1M1
	$\log_3(x-5)+\log_3(x+3)=2 \Rightarrow \log_3\left[(x-5)(x+3)\right]=2$	M1
	$\Rightarrow (x-5)(x+3)=3^2 \Rightarrow x^2-2x-24=0$	M1A1
	$(x+4)(x-6)=0 \Rightarrow x=6$ (reject $x=-4$)	dM1A1
		[7]
		Total 7 marks

Mark	Notes
Method 1 – Works in base 3	
M1	Uses $n\log A=\log A^n$ correctly to write $\log_3\sqrt{x-5}=\frac{1}{2}\log_3(x-5)$
M1	For an attempt to change the base of $\log_9(x+3)$ to base 3 using $\log_a x=\frac{\log_b x}{\log_b a}$ $\log_9(x+3)=\frac{\log_3(x+3)}{\log_3 9}=\frac{\log_3(x+3)}{2}$ [accept $\frac{\log_3(x+3)}{p}$ where $p\neq 1$]
M1	Uses $\log A+\log B=\log AB$ to correctly combine the logs $\log_3(x-5)+\log_3(x+3)=\log_3(x-5)(x+3)$
M1	For removing the logs in the equation to obtain $(x-5)(x+3)=3^2$ and rearranging to a 3TQ
A1	For obtaining a correct 3TQ. $x^2-2x-24=0$
Method 2 – Works in base 9	
M1	Uses $n\log A=\log A^n$ correctly to write $\log_3\sqrt{x-5}=\frac{1}{2}\log_3(x-5)$
M1	For an attempt to change the base of $\log_3\sqrt{x-5}$ or $\frac{1}{2}\log_3(x-5)$ to base 9 using $\log_a x=\frac{\log_b x}{\log_b a}$ $\log_3\sqrt{x-5}=\frac{\log_9\sqrt{x-5}}{\log_9 3}=\frac{\log_9\sqrt{x-5}}{1/2}=2\log_9\sqrt{x-5}$ [accept $q\log_9\sqrt{x-5}$ where q$\neq$ 1] $\frac{1}{2}\log_3(x-5)=\frac{1}{2}\times\frac{\log_9(x-5)}{\log_9 3}=\frac{1}{2}\times\frac{\log_9(x-5)}{1/2}=\log_9(x-5)$
M1	Uses $\log A+\log B=\log AB$ to correctly combine the logs $\log_9(x-5)+\log_9(x+3)=\log_9(x-5)(x+3)$
M1	For removing the logs in the equation to obtain $(x-5)(x+3)=9$ and rearranging to a 3TQ
A1	For obtaining a correct 3TQ. $x^2-2x-24=0$
Attempt to solve the quadratic equation	
dM1	For an attempt to solve their 3TQ. See General Guidance for the definition of an attempt. Dependent on at least one previous M mark scored.
A1	$x=6$ Must reject $x=-4$ if this solution is also included.

Question number	Scheme	Marks
6	$\left[\frac{\mathrm{d}V}{\mathrm{d}t}=3\ \left(\mathrm{cm}^3/\mathrm{s}\right)\right]$ $V=\frac{4}{3}\pi r^3\quad \frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2,\ \ A=4\pi r^2\quad \frac{\mathrm{d}A}{\mathrm{d}r}=8\pi r$ $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}$ oe $\frac{\mathrm{d}A}{\mathrm{d}t}=[8\pi\times 10]\times\left[\frac{1}{4\pi\times 10^2}\right]\times 3=0.6\ (\mathrm{cm}^2/s)$	M1A1,A1 (M1 for any one) M1 dM1A1 [6]
		Total 6 marks

Mark	Notes
M1	For using the **correct** formula for volume of a sphere or for surface area of a sphere and attempt to differentiate their expression. [See General Guidance for definition of attempt to differentiate]
A1	For one correct $\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$ or $\frac{\mathrm{d}A}{\mathrm{d}r}=8\pi r$
A1	For both correct $\frac{\mathrm{d}V}{\mathrm{d}r}=4\pi r^2$ and $\frac{\mathrm{d}A}{\mathrm{d}r}=8\pi r$
M1	For applying a **correct** Chain rule using their $\frac{\mathrm{d}V}{\mathrm{d}r}$, their $\frac{\mathrm{d}A}{\mathrm{d}r}$ and $\frac{\mathrm{d}V}{\mathrm{d}t}=3$ to obtain $\frac{\mathrm{d}A}{\mathrm{d}t}=\frac{\mathrm{d}A}{\mathrm{d}r}\times\frac{\mathrm{d}r}{\mathrm{d}V}\times\frac{\mathrm{d}V}{\mathrm{d}t}='\,8\pi r'\times'\frac{1}{4\pi r^2}'\times 3$ May be seen in two stages.
dM1	For substitution of $r=10$ into their expression for $\frac{\mathrm{d}A}{\mathrm{d}t}$ to obtain $\frac{\mathrm{d}A}{\mathrm{d}t}='\,8\pi\times 10'\times'\frac{1}{4\pi\times 10^2}'\times 3$
A1	$\frac{\mathrm{d}A}{\mathrm{d}t}=0.6\ (\mathrm{cm}^2\ /\ \mathrm{s})$

Question number	Scheme	Marks
7 (a)	**Throughout this question condone missing degree signs**	
	$\cos\theta^{\circ}=\frac{6^2+8^2-k^2}{2\times6\times8}=\frac{100-k^2}{96}$ *	M1A1cso [2]
(b)	$\sqrt{455}=\frac{1}{2}\times6\times8\times\sin\theta^{\circ}$	M1
	$\Rightarrow\sin\theta^{\circ}=\frac{\sqrt{455}}{24}\Rightarrow\left(\sin^2\theta^{\circ}=\frac{455}{576}\right)$	A1
	$\cos^2\theta^{\circ}=1-\sin^2\theta^{\circ}\Rightarrow\cos^2\theta^{\circ}=1-\frac{455}{576}=\frac{121}{576}$	M1
	$\Rightarrow\cos\theta^{\circ}=\pm\frac{11}{24}$ both values required	A1
	$\cos\theta^{\circ}=\frac{11}{24}=\frac{100-k^2}{96}\Rightarrow k^2=56\Rightarrow k=\sqrt{56}=\left(2\sqrt{14}\right)$	M1A1
	$\cos\theta^{\circ}=-\frac{11}{24}=\frac{100-k^2}{96}\Rightarrow k^2=144\Rightarrow k=12$	A1
		[7]
	ALT $\sqrt{455}=\frac{1}{2}\times6\times8\times\sin\theta^{\circ}$	[M1
	$\Rightarrow\sin\theta^{\circ}=\frac{\sqrt{455}}{24}$	A1
	$\theta^{\circ}=\sin^{-1}\left(\frac{\sqrt{455}}{24}\right)\ (=62.72\ldots^{\circ})$	M1
	$\theta^{\circ}=62.72\ldots,117.27\ldots$ both values required	A1
	$\cos\theta^{\circ}=\frac{100-k^2}{96}\Rightarrow k^2=100-96\cos\theta^{\circ}$ $\Rightarrow k^2=100-96\cos 62.72\ldots\Rightarrow k^2=56\Rightarrow k=\sqrt{56}$	M1A1
	$\cos\theta^{\circ}=\frac{100-k^2}{96}\Rightarrow k^2=100-96\cos\theta^{\circ}$ $\Rightarrow k^2=100-96\cos 117.27\ldots\Rightarrow k^2=144\Rightarrow k=12$	A1]
		Total 9 marks

Part	Mark	Notes
(a)	**M1**	For correct substitution into the cosine rule and attempt to rearrange to find an expression for $\cos\theta^{\circ}$
	A1 **cso**	For obtaining the given expression for $\cos\theta$ $\cos\theta^{\circ}=\frac{100-k^2}{96}$ **Note: This is a show question. There must be no errors seen.**

(b)	**M1**	For using the **correct** formula for area of a triangle and substitution of the given values to obtain $\sqrt{455} = \frac{1}{2} \times 6 \times 8 \times \sin\theta°$ and attempt to rearrange to obtain $\sin\theta° = \cdots$
	A1	$\sin\theta° = \frac{\sqrt{455}}{24}$ Allow for $\sin\theta = \frac{\sqrt{455}}{0.5\times 8\times 6}$
	M1	For use of $\sin^2\theta + \cos^2\theta = 1$ to obtain a value for $\cos\theta°$. $\cos^2\theta° = 1 - \frac{'455'}{576} \Rightarrow \cos\theta° = \pm\sqrt{1 - \frac{'455'}{576}}$ Allow use of their $\sin\theta°$ provided $-1 \leq \sin\theta° \leq 1$ Allow if only one value of $\cos\theta°$ obtained.
	A1	$\cos\theta° = \pm\frac{11}{24}$
	M1	For forming an equation for *k* using their $\cos\theta°$ and attempt to solve for *k*. $\cos\theta° = \frac{'11'}{24} = \frac{100-k^2}{96} \Rightarrow k^2 = 56 \Rightarrow k = \sqrt{56}$ $\cos\theta° =' -\frac{11'}{24} = \frac{100-k^2}{96} \Rightarrow k^2 = 144 \Rightarrow k = 12$
	A1	For one correct value of *k* $\sqrt{56}$ or awrt 7.48 **or** awrt 12
	A1	For both correct values of *k* $\sqrt{56}$ or awrt 7.48 **and** awrt 12
ALT – working with angles		
	M1	For using the **correct** formula for area of a triangle and substitution of the given values to obtain $\sqrt{455} = \frac{1}{2} \times 6 \times 8 \times \sin\theta°$ and attempt to rearrange to obtain $\sin\theta° = \cdots$
	A1	$\sin\theta° = \frac{\sqrt{455}}{24}$ Allow for $\sin\theta = \frac{\sqrt{455}}{0.5\times 8\times 6}$
	M1	For use of the inverse trigonometric function to obtain a value for $\theta°$ $\theta° = 62.72\ldots, 117.27\ldots$ If working not shown then award for angle correct to a minimum of 1 d.p. Allow if only one value of $\theta°$ found. Condone working in radians awrt 1.09, awrt 2.05
	A1	$\theta° = 62.72\ldots, 117.27\ldots$ Allow awrt 62.7°, 117.3° Both angles found. Condone working in radians awrt 1.09, awrt 2.05
	M1	For forming an equation in *k* using their θ and an attempt to solve for k. $\cos\theta° = \frac{100-k^2}{96} \Rightarrow k^2 = 100 - 96\cos\theta°$ $\Rightarrow k^2 = 100 - 96\cos 62.72\ldots \Rightarrow k^2 = 56 \Rightarrow k = \sqrt{56}$ $\cos\theta° = \frac{100-k^2}{96} \Rightarrow k^2 = 100 - 96\cos\theta°$ $\Rightarrow k^2 = 100 - 96\cos 117.27\ldots \Rightarrow k^2 = 144 \Rightarrow k = 12$
	A1	For one correct value of *k* $\sqrt{56}$ or awrt 7.48 **or** awrt 12
	A1	For both correct values of *k* and no others. $\sqrt{56}$ or awrt 7.48 **and** awrt 12

Question number	Scheme	Marks
8 (a)	$\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{AB}=\mathbf{a}+\mathbf{b}$	B1 [1]
(b)	$\overrightarrow{OC}=2\mathbf{b}$ $\overrightarrow{BC}=\overrightarrow{BO}+\overrightarrow{OC}=-(\mathbf{a}+\mathbf{b})+2\mathbf{b}=\mathbf{b}-\mathbf{a}$	B1 M1A1 [3]
(c)	$\overrightarrow{OM}=\overrightarrow{OB}+\overrightarrow{BM}=\mathbf{a}+\mathbf{b}+\frac{2}{3}(\mathbf{b}-\mathbf{a})=\frac{\mathbf{a}}{3}+\frac{5\mathbf{b}}{3}$ or $\frac{1}{3}(\mathbf{a}+5\mathbf{b})$	M1A1ft [2]
(d)	$\overrightarrow{OY}=\mu\left(\frac{\mathbf{a}}{3}+\frac{5\mathbf{b}}{3}\right)=\frac{\mu\mathbf{a}}{3}+\frac{5\mu\mathbf{b}}{3}$ $\overrightarrow{OY}=\overrightarrow{OA}+\overrightarrow{AY}=\boldsymbol{a}+\lambda\boldsymbol{b}$ $\Rightarrow\frac{\mu}{3}=1$ and $\frac{5\mu}{3}=\lambda$ Solves simultaneous equations by any method $\mu=3,\ \lambda=5$ $AB:BY=1:4$ **ALT** $\overrightarrow{BY}=\lambda\overrightarrow{AB}=\lambda\mathbf{b}$ $\overrightarrow{BY}=\overrightarrow{BO}+\overrightarrow{OY}=\overrightarrow{BO}+\mu\overrightarrow{OM}=-(\boldsymbol{a}+\mathbf{b})+\mu\left(\frac{1}{3}\mathbf{a}+\frac{5}{3}\boldsymbol{b}\right)$ $=\left(-1+\frac{1}{3}\mu\right)\mathbf{a}+\left(-1+\frac{5}{3}\mu\right)\mathbf{b}$ $\Rightarrow-1+\frac{1}{3}\mu=0$ and $\lambda=-1+\frac{5}{3}\mu$ $\mu=3,\lambda=4$ $AB:BY=1:4$	M1 M1 M1 M1 A1 [5] [M1 M1 M1 M1 A1]
(e)	$10=\frac{1}{2}ab\sin 60^{\circ}\Rightarrow ab=\frac{40}{\sqrt{3}}\Rightarrow a=\frac{40}{b\sqrt{3}}$ Area $=\frac{1}{2}a\times5b\sin120^{\circ}=\frac{1}{2}\times\frac{40}{b\sqrt{3}}\times5b\sin120^{\circ}$ Area $=50$ **ALT** $\frac{\text{Area }OAY}{\text{Area }OAB}=\frac{\frac{1}{2}\times h\times5}{\frac{1}{2}\times h\times1}=\frac{5}{1}$ Area $OAY=5\times$ Area OAB Area $OAB=10$ Area $=50$	M1A1 dM1 A1 [4] [M1 A1 dM1 A1]
		Total 15 marks

Part	Mark	Notes
(a)	**B1**	For a correct expression for $\overrightarrow{OB}$ in terms of **a** and **b**
(b)	**B1**	For a correct expression for $\overrightarrow{OC}$
	M1	For a correct vector statement for $\overrightarrow{BC}$: $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$ This mark can be implied by a correct (unsimplified) vector using their $\overrightarrow{OB}$. Vector statement must be suitable for substitution to find $\overrightarrow{BC}$
	A1	For the correct simplified $\overrightarrow{BC}$ in terms of a single **a** and **b** only. $\overrightarrow{BC} = \mathbf{b} - \mathbf{a}$ If answer $\overrightarrow{BC} = \mathbf{b} - \mathbf{a}$ seen without wrong working then award B1M1A1.
(c)	**M1**	For a correct vector statement for $\overrightarrow{OM}$: $\overrightarrow{OM} = \overrightarrow{OB} + \frac{2}{3}\overrightarrow{BC}$ This mark can be implied by a correct (unsimplified) vector using their $\overrightarrow{OB}$ and their $\overrightarrow{BC}$.
	A1ft	For the correct simplified using their $\overrightarrow{OM}$ in terms of a single **a** and **b** only. $\overrightarrow{OM} = \frac{\boldsymbol{a}}{3} + \frac{5\boldsymbol{b}}{3}$ or $\frac{1}{3}(\boldsymbol{a} + 5\boldsymbol{b})$
(d)	**M1** **M1**	M1 for one correct statement of route for $\overrightarrow{OY}$ M1 for second correct statement of route for $\overrightarrow{OY}$ $\overrightarrow{OY} = \mu\overrightarrow{OM}$ (or any other variable in place of μ) $\overrightarrow{OY} = \left(\overrightarrow{OA} + \overrightarrow{AY}\right) = \mathbf{a} + \lambda\mathbf{b}$ (or any other variable in place of λ, provided this is different to their μ). Allow use of their vectors from earlier parts of the question.
	M1	For equating their coefficients of **a** and **b** to obtain two equations. Mark intent – one must be correct, condone slips in second.
	M1	Solving their simultaneous equations by any method. Only the value for their λ is required for this mark.
	A1	For $AB: BY = 1: 4$
ALT – use of $\overrightarrow{BY}$ route, use also for $\overrightarrow{AY}$ route		
	M1 **M1**	M1 for one correct statement of route for $\overrightarrow{BY}$ M1 for second correct statement of route for $\overrightarrow{BY}$ $\overrightarrow{BY} = \lambda\overrightarrow{AB} = \lambda\mathbf{b}$ (or any other variable in place of λ) $\overrightarrow{BY} = \overrightarrow{BO} + \overrightarrow{OY} = \overrightarrow{BO} + \mu\overrightarrow{OM} = -(\boldsymbol{a} + \mathbf{b}) + \mu\left(\frac{1}{3}\mathbf{a} + \frac{5}{3}\boldsymbol{b}\right)$ $= \left(-1 + \frac{1}{3}\mu\right)\mathbf{a} + \left(-1 + \frac{5}{3}\mu\right)\mathbf{b}$ (or any other variable in place of μ provided this is different to their λ). Allow use of their vectors from earlier parts of the question.
	M1	For equating their coefficients of **a** and **b** to obtain two equations. Mark intent – one must be correct, condone slips in second.
	M1	Solving their simultaneous equations by any method. Only the value for their λ is required for this mark.
	A1	For $AB: BY = 1: 4$

(e)	**M1**	For use of the **correct** formula for area of a triangle with 60° and correct value of sin 60° $10 = \frac{1}{2}ab\sin 60^\circ = ab \times \frac{\sqrt{3}}{4}$
	A1	For correct expression for *ab* or *a* $ab = \frac{40}{\sqrt{3}}$ or $a = \frac{40}{b\sqrt{3}}$
	dM1	For use of the **correct** formula for area of a triangle with 120° and attempt to substitute for *ab* or *a*. Area $= \frac{1}{2}a \times {'5'}b\sin 120^\circ = \frac{1}{2} \times {'}\frac{40}{b\sqrt{3}}{'} \times {'5'}b\sin 120^\circ$ or Area $= \frac{1}{2}a \times {'5'}b\sin 120^\circ = \frac{1}{2} \times ab \times {'5'}\sin 120^\circ = \frac{1}{2} \times {'}\frac{40}{\sqrt{3}}{'} \times {'5'}\sin 120^\circ$ Dependent on the first M awarded. Allow use of their λ from part (d).
	A1	For the correct area Area $= 50$
ALT – use of ratios of areas		
	M1	For use of their ratio $AB{:}BY$ to write an equation linking area *OAY* and area *OAB* $\frac{\text{Area } OAY}{\text{Area } OAB} = \frac{\frac{1}{2}\times h\times(1+{'4'})}{\frac{1}{2}\times h\times 1} = \frac{'5'}{1}$
	A1	For correct relationship between area *OAY* and area *OAB*
	dM1	For a correct method to find the area of *OAB* Dependent on first M mark being awarded.
	A1	For the correct area Area $= 50$

Question number	Scheme	Marks
9 (a)	$a=5\times1-1=4$ $d=5$ $S_n=\frac{n}{2}\left(2\times4+[n-1]5\right)=\frac{n}{2}\left(3+5n\right)*$ cso **ALT** $a=4$ $l=5n-1$ $S_n=\frac{n}{2}(a+l)=\frac{n}{2}(4+5n-1)=\frac{n}{2}(3+5n)*$ cso	B1 M1A1 [3] [B1 M1A1]
(b)	$\sum_{r=10}^{20}(5r-1)=\sum_{r=1}^{20}(5r-1)-\sum_{r=1}^{9}(5r-1)$ $\sum_{r=10}^{20}(5r-1)=\frac{20}{2}(3+5\times20)-\frac{9}{2}(3+5\times9)$ $=1030-216=814$ **ALT** $a=4+9\times5=49,\ l=4+20\times5=99,\ n=20-10+1=11$ $\sum_{r=10}^{20}(5r-1)=\frac{11}{2}(49+99)=814$ **ALT** $a=5\times10-1=49$ $d=5$ $n=11$ $S_n=\frac{11}{2}(2\times49+(11-1)\times5)=814$	B1 M1 A1 [3] [B1 M1A1] [B1 M1A1]
(c)	$\frac{n}{2}(3+5n)=12(4+5n)+52\Rightarrow5n^2-117n-200=0$ $\Rightarrow(n-25)(5n+8)=0\Rightarrow n=25,\ \left(n\neq-\frac{8}{5}\right)$	M1M1A1 M1A1 [5]
		Total 11 marks

Part	Mark	Notes
(a)	**B1**	For finding the first term and common difference. $a=5\times1-1=4$ $d=5$ May be implied by correct values seen in summation formula.
	M1	Uses a **correct** form of the summation formula for an arithmetic series with their *a* and their *d* provided their *a* and their *d* are stated. $S_n=\frac{n}{2}(2\times'4'+[n-1]\times'5')$
	A1 **cso**	For obtaining the given answer in full with no errors. $\sum_{r=1}^{n}(5r-1)=\frac{n}{2}(3+5n)$
Alternative method		
	B1	For finding the first term and an expression for the last term. $a=4$ $l=5n-1$

		May be implied by correct values seen in summation formula.
	M1	Uses a **correct** form of the summation formula for an arithmetic series with their *a* and their *l* provided their *a* and their *l* are stated. $S_n = \frac{n}{2}(a + l) = \frac{n}{2}('4' + '5n - 1')$
	A1 **cso**	For obtaining the given answer in full with no errors. $\sum_{r=1}^{n}(5r - 1) = \frac{n}{2}(3 + 5n)$
Note: If standard summation results are correctly used award B1M1A1, if not fully correct send to review.		
(b)	**B1**	For correctly giving the required summation as the difference between two summations starting at $r = 1$. $\sum_{r=10}^{20}(5r - 1) = \sum_{r=1}^{20}(5r - 1) - \sum_{r=1}^{9}(5r - 1)$
	M1	For substitution of $n = 20$ and $n = '9'$ into the result from part (a) and subtracting. $\frac{20}{2}(3 + 5 \times 20) - \frac{'9'}{2}(3 + 5 \times' 9')$ Allow for use of 9 or 10.
	A1	For the correct summation 814
Alternative method		
	B1	For finding the first term, last term and number of terms for the arithmetic sequence. $a = 49, l = 99, n = 11$
	M1	Uses a **correct** summation formula for an arithmetic series with their *a*, their $l$and their n provided these are stated. $\frac{'11'}{2}('49' + '99')$
	A1	For the correct summation 814
Alternative method – considering this as a series starting at the 10th term of the original series		
	B1	For finding the first term, common difference and number of terms. $a = 5 \times 10 - 1 = 49$ $d = 5$ $n = 11$
	M1	Uses a **correct** form of the summation formula for an arithmetic series with their *a*, *their d*, and *their n* provided their *a* and their *d* and their *n* are stated and their *n*≠20 $S_n = \frac{'11'}{2}(2 \times '49' + ('11' - 1) \times '5')$
	A1	For the correct summation 814
(c)	**M1**	Uses $5r - 1$ with $n + 1$ to find an expression for u_{n+1} in terms of *n*. $5r - 1 = 5(n + 1) - 1 = 5n + 4$
	M1	Forms a correct equation for *n* using the result given in part (a) and their expression for $5r - 1$ in terms of *n*. $\frac{n}{2}(3 + 5n) = 12('4 + 5n') + 52$
	A1	Obtains a correct 3TQ $5n^2 - 117n - 200 = 0$ oe
	M1	For an attempt to solve their 3TQ. See General Guidance for the definition of an attempt. $(n - 25)(5n + 8) = 0 \;\Rightarrow\; n = 25, \left(n = -\frac{8}{5}\right)$
	A1	For correct value: $n = 25$ If $n = -\frac{8}{5}$ is seen it must be rejected.

Question number	Scheme	Marks
10 (a)	$\frac{1}{2}+\sin 3x=0\Rightarrow\sin 3x=-\frac{1}{2}\Rightarrow 3x=-\frac{\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}\Rightarrow x=\frac{7\pi}{18},\frac{11\pi}{18}$	M1
	Coordinates of M are $\left(\frac{7\pi}{18},\ 0\right)$*	A1
	Coordinates of N are $\left(\frac{11\pi}{18},\ 0\right)$	A1 [3]
	ALT	
	$\frac{1}{2}+\sin\left(3\times\frac{7\pi}{18}\right)=0$	[M1
	Coordinates of M are $\left(\frac{7\pi}{18},0\right)*$	A1
	Coordinates of N are $\left(\frac{11\pi}{18},0\right)$	A1]
(b)	$\frac{dy}{dx}=3\cos 3x=0\Rightarrow 3x=\frac{\pi}{2}\Rightarrow x=\frac{\pi}{6}$	M1A1
	$y=\frac{1}{2}+\sin 3\left('\frac{\pi}{6}'\right)=1.5$	dM1A1 [4]
	$\left[\text{Coordinates of point } A \text{ are } \left(\frac{\pi}{6},\ 1.5\right)\right]$	
	ALT	
	Max of sine curve is 1 so that $\frac{1}{2}+1=\frac{3}{2}\Rightarrow y=\frac{3}{2}$	[M1A1
	$\sin 3\theta=1\ \Rightarrow\ 3\theta=\frac{\pi}{2}\ \Rightarrow\ \theta=\frac{\pi}{6}$	dM1A1]
	$\left[\text{Coordinates of point } A \text{ are } \left(\frac{\pi}{6},\ \frac{3}{2}\right)\right]$	
(c)	Uses the given x coordinate for point M	
	At point M $\quad\frac{dy}{dx}=3\cos\left(3\times\frac{7\pi}{18}\right)=-\frac{3\sqrt{3}}{2}$	B1
	$y-0=-\frac{3\sqrt{3}}{2}\left(x-\frac{7\pi}{18}\right)\Rightarrow 12y+18\sqrt{3}x-7\sqrt{3}\pi=0$ o.e.	M1A1A1 [4]
(d)	$A=\int_0^{\frac{7\pi}{18}}\left(\frac{1}{2}+sin\,3\,x\right)\,dx+\left\lVert\left[\int_{\frac{7\pi}{18}}^{'\frac{11\pi}{18}'}\left(\frac{1}{2}+sin\,3\,x\right)\,dx\right]\right\rVert$	M1
	$A=\left[\frac{1}{2}x-\frac{cos\,3\,x}{3}\right]_0^{\frac{7\pi}{18}}+\left\lVert\left[\frac{1}{2}x-\frac{cos\,3\,x}{3}\right]\right\rVert_{\frac{7\pi}{18}}^{'\frac{11\pi}{18}'}$	M1
	$A=\left[\left(\frac{1}{2}\times\frac{7\pi}{18}-\frac{cos\,3\left(\frac{7\pi}{18}\right)}{3}\right)-\left(0-\frac{cos\,3\times 0}{3}\right)\right]$	
		M1

	$+\left\lvert\left[\left(\frac{1}{2}\times\frac{11\pi}{18}-\frac{cos\,3\left(\frac{'11\pi'}{18}\right)}{3}\right)-\left(\frac{1}{2}\times\frac{7\pi}{18}-\frac{cos\,3\left(\frac{7\pi}{18}\right)}{3}\right)\right]\right\rvert$ $A=[(1.23287)+(0.22828)]=1.46115\ldots\approx 1.46$	A1 [4]
		Total 15 marks

Part	Mark	Notes
(a)	**M1**	Sets the equation equal to 0, solves using inverse sin and obtains a correct angle $\frac{1}{2}+\sin 3x=0\ \Rightarrow\ \sin 3x=-\frac{1}{2}\ \Rightarrow\ 3x=-\frac{\pi}{6}$ Condone working in degrees for M mark.
	A1 **cso**	For correctly obtaining $\left(\frac{7\pi}{18},0\right)$ * with no errors. Award for finding $\frac{7\pi}{18}$ and $\frac{11\pi}{18}$ but not shown as coordinates.
	A1	For correct coordinates of N: $\left(\frac{11\pi}{18},0\right)$
Alternative method		
	M1	For correct substitution of $\frac{7\pi}{18}$ into $\frac{1}{2}+\sin 3x$
	A1 **cso**	For correctly showing that $\frac{1}{2}+\sin\left(3\times\frac{7\pi}{18}\right)=0$ and stating $\left(\frac{7\pi}{18},0\right)$ * Award for showing $\frac{7\pi}{18}$ and finding $\frac{11\pi}{18}$ but not shown as coordinates
	A1	For correct coordinates of N: $\left(\frac{11\pi}{18},0\right)$
(b)	**M1**	For attempt to differentiate $y=\frac{1}{2}+\sin 3x$, set equal to 0 and solve for *x*. $\frac{\mathrm{d}y}{\mathrm{d}x}=3\cos 3x=0\ \Rightarrow\ 3x=\frac{\pi}{2}\ \Rightarrow\ x=\frac{\pi}{6}$ Condone finding *x* in degrees for this mark. Attempt at differentiation of two terms: $\frac{1}{2}\to 0$ $\sin(3x)\to k\cos(3x)$
	A1	For correctly obtaining $x=\frac{\pi}{6}$ **Note: Do not award this mark if $\frac{\mathrm{d}y}{\mathrm{d}x}$ is incorrect.**
	dM1	Substitutes $x=\frac{\pi}{6}$ to find a value for *y*. $y=\frac{1}{2}+\sin 3\left('\frac{\pi}{6}'\right)=1.5$ Condone working with *x* in degrees for this mark.
	A1	For correct coordinates of *A*. $\left(\frac{\pi}{6},\frac{3}{2}\right)$ Allow values equivalent to $\frac{3}{2}$
Alternative method		
	M1	For stating that the maximum of a sine curve is 1 and adding $\frac{1}{2}$
	A1	For correctly obtaining $y=\frac{3}{2}$
	dM1	For setting $\sin 3x$ equal to 1 and attempt to solve for *x* $\sin 3x=1\ \Rightarrow\ 3x=\frac{\pi}{2}\ \Rightarrow\ x=\frac{\pi}{6}$ Condone working with *x* in degrees for this mark.

	A1	For correctly obtaining $x = \frac{\pi}{6}$
(c)	**B1**	For correct gradient at point *M* $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = -\frac{3\sqrt{3}}{2}$
	M1	For a fully correct method of finding the equation of the tangent to *C* at *M* $y - 0 = ' - \frac{3\sqrt{3}}{2}' \left(x - \frac{7\pi}{18}\right)$ Allow use of their $-\frac{3\sqrt{3}}{2}$ obtained from substitution of $x = \frac{7\pi}{18}$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	A1	For a correct equation of the tangent in any form: $y - 0 = -\frac{3\sqrt{3}}{2}\left(x - \frac{7\pi}{18}\right)$
	A1	For correct equation of the tangent in the required form: $12y + 18\sqrt{3}x - 7\sqrt{3}\pi = 0$ Accept integer multiples of this equation.
(d)	**M1**	For identifying the correct limits to find the area above the curve and the area below the curve: $\int_0^{\frac{7\pi}{18}}\left(\frac{1}{2} + \sin 3x\right) + \left\|\int_{\frac{7\pi}{18}}^{\frac{11\pi}{18}}\left(\frac{1}{2} + \sin 3x\right)\right\|$ or $\int_0^{\frac{7\pi}{18}}\left(\frac{1}{2} + \sin 3x\right) - \int_{\frac{7\pi}{18}}^{\frac{11\pi}{18}}\left(\frac{1}{2} + \sin 3x\right)$ Allow use of their $\frac{11\pi}{18}$. Needs to correctly indicate dealing with areas above and below axis.
	M1	For an attempt to integrate $\frac{1}{2} + \sin 3x$ obtaining: $\frac{1}{2}x - k\cos 3x \quad \text{where } k \neq -3$ For this mark ignore incorrect / absent limits.
	M1	For substituting limits correctly into their integrated expression (must be a changed expression). $A = \left[\left(\frac{1}{2} \times \frac{7\pi}{18} - \frac{cos\, 3\left(\frac{7\pi}{18}\right)}{3}\right) - \left(0 - \frac{cos\, 3 \times 0}{3}\right)\right]$ $+ \left\|\left[\left(\frac{1}{2} \times \frac{11\pi}{18} - \frac{cos\, 3\left(\frac{'11\pi'}{18}\right)}{3}\right) - \left(\frac{1}{2} \times \frac{7\pi}{18} - \frac{cos\, 3\left(\frac{7\pi}{18}\right)}{3}\right)\right]\right\|$ Allow use of their $\frac{11\pi}{18}$. Must show substitution or M0.
	A1	For correctly obtaining awrt 1.46

Question number	Scheme	Marks
11 (a)	$\text{f}(x)=\int ax^2-14x-10\ \text{d}x=\frac{ax^3}{3}-\frac{14x^2}{2}-10x+c$	M1A1
	$\text{f}(4)=\frac{a\times4^3}{3}-\frac{14\times4^2}{2}-10\times4+c=0\Rightarrow\frac{64a}{3}-152+c=0$	M1
	$\text{f}(-1)=\frac{a\times(-1)^3}{3}-\frac{14\times(-1)^2}{2}-10\times(-1)+c=25\Rightarrow-\frac{a}{3}-22+c=0$	M1
	$\frac{64a}{3}-152+c=0$	
	$-\frac{a}{3}-22+c=0$	
	$\Rightarrow152-\frac{64a}{3}=22+\frac{a}{3}\Rightarrow a=6$	M1A1 [6]
(b)	$c=22+\frac{6}{3}=24$	B1
	$\text{f}(x)=2x^3-7x^2-10x+24$	
	$\begin{array}{r}2x^2+x-6\\ x-4\overline{\smash{)}2x^3-7x^2-10x+24}\end{array}$	M1A1
	$\text{f}(x)=(x-4)(2x-3)(x+2)=0$	dM1A1
	$x=4,\ \frac{3}{2},\ -2$	A1 [6]
	ALT $2x^3-7x^2-10x+24=(x-4)(ax^2+bx+c)$ $a=2, b=1, c=-6$	[M1
	$f(x)=(x-4)(2x^2+x-6)$	A1
	$f(x)=(x-4)(2x-3)(x+2)=0$	dM1A1
	$x=4,\frac{3}{2},-2$	A1]
		Total 12 marks

Part	Mark	Notes
(a)	**M1**	For an attempt to integrate $\text{f}'(x)=ax^2-14x-10$ See General Guidance on what constitutes an attempt to integrate.
	A1	For correctly integrating $\text{f}'(x)$ to obtain $\text{f}(x)=\frac{ax^3}{3}-\frac{14x^2}{2}-10x+c$ Must include $+c$ for this mark.
	M1	For substituting $x=\pm4$ in their $\text{f}(x)$ and setting =0
	M1	For substituting $x=\pm1$ in their $\text{f}(x)$ and setting =25

	M1	For correct method to solve the equations simultaneously to find a A correct intermediate step is required e.g. $65a = 390$
	A1 **cso**	For fully correct working leading to $a = 6$
(b)	**B1**	For correctly identifying $c = 24$
	M1	For attempt to divide f(x) by $x - 4$ Must get as far as $2x^2 + \cdots$
	A1	For correct division of f(x) by $x - 4$ to obtain $2x^2 + x - 6$
	dM1	For an attempt to factorise their 3TQ which must come from a cubic. See General Guidance on what constitutes an attempt to factorise.
	A1	For obtaining correct factorisation of the cubic: $(x - 4)(2x - 3)(x + 2)$
	A1	For all three correct solutions of the equation: $x = 4, \frac{3}{2}, -2$
Alternative method		
	M1	For an attempt to find the quadratic factor that multiplies $x - 4$ to give f(x) Must get as far as $f(x) = (x - 4)(2x^2 + bx + c)$
	A1	For correctly comparing coefficients to obtain $2x^2 + x - 6$
	dM1	For an attempt to factorise their 3TQ which must come from a cubic. See General Guidance on what constitutes an attempt to factorise.
	A1	For obtaining correct factorisation of the cubic: $(x - 4)(2x - 3)(x + 2)$
	A1	For all three correct solutions of the equation: $x = 4, \frac{3}{2}, -2$
	Note: Correct solution seen with no working scores B0M0A0M0A0A0	

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